On accuracy of averaging in plane stick-slip problems and tectonic earthquake modeling

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1. Tectonic earthquakes are caused by displacement breaks (DB) at large distances along active faults (AF), which are the layers of thickness h_F~100 m with strongly destructed (fissured) fluid-filled rocks, with dilatancy and microcracks interaction. Actually, large DBs occur in much thinner ultracataclastic layers of h_u~1-10 mm thickness. So, at h_T>>h_F scales tectonic blocks (TB, zones outside AF) can be considered as continuous elastic (or elastic-viscous) media (EM), macrocracks can be considered as plane, and conditions inside AFs, regulating stick-slip processes, as boundary conditions at slip surface. For chains of TBs of L_T>>h_T scale various models of chains of movement equations of point masses (EPM) are typically used, interacting one with another and with state parameters inside AF-DB, such as Barridge-Knopoff model with Rate&State friction laws, and others. The question arises: how far EPM solutions differ from exact solutions of corresponding initial-boundary value problems?

2. Let's consider that EM inside TB is linear isotropic homogeneous flat and has only one component of tectonic displacements $u_i = \delta_{i1} u(x^{(1)}, x^{(2)}, t)$ along AF ($x^{(2)}$ – across AF). Then

$$u = u^{(1)}(x^{(1)}, t) + u^{(2)}(x^{(2)}, t),\ {}^{(k)}r^2 u^{(k)} = u^{(k)}_{,kk},\ (k=1,2), \qquad (1)$$

where ${}^{(1)}r \equiv r = \partial_t$, ${}^{(2)}r = {}^{(1)}rq$, $q = \tau^{(2)}/\tau^{(1)}$, $\tau^{(k)} = L^{(k)}/c^{(k)}$, $c^{(1)} = \sqrt{\dfrac{\Lambda + 2G}{\rho}}$ and $c^{(2)} = \sqrt{\dfrac{G}{\rho}}$, $L^{(1)}$ and $L^{(2)}$ are longitudinal and transverse velocities and TB dimensions (dimensionless t, $x^{(1)}$, $x^{(2)}$ measured in $\tau^{(1)}$, $L^{(1)}$, $L^{(2)}$). One (1) leads to 6 equations

$$V^{(k)\pm} = \pm\frac{1}{N^{(k)}}(M^{(k)}u^{(k)\pm} - \bar{u}^{(k)}),\ {}^{(k)}r^2\bar{u}^{(k)} = V^{(k)+} - V^{(k)-} \qquad (2)$$

for 10 values $u^{(k)\pm} = u^{(k)}(1/0, t)$, $V^{(k)\pm} = u^{(k)}_{,k}(1/0, t)$, $\bar{u}^{(k)} = \int_0^1 u^{(k)}(x^{(k)}, t)dx^{(k)}$, where $M^{(k)} = M({}^{(k)}r)$, $M(r) = \mathrm{sh}(r)/\mathrm{r}$, $N(r) = (\mathrm{ch}(r) - 1)/r^2$. For (1), boundary conditions are 4 relations for surface-averaged displacements $\bar{u}^{(1)\pm} = u^{(1)\pm} + \bar{u}^{(2)}$, $\bar{u}^{(2)\pm} = u^{(2)\pm} + \bar{u}^{(1)}$ and stresses $\bar{\sigma}^{(1)\pm}_{11} = R^{(1)}V^{(1)\pm}$, $\bar{\sigma}^{(2)\pm}_{12} = R^{(2)}V^{(2)\pm}$ $\bar{\sigma}^{(2)\pm}_{22} = R^{(22)}(u^{(1)+} - u^{(1)-})$, where $R^{(1)} = (\Lambda + 2G)/L^{(1)}$, $R^{(2)} = G/L^{(2)}$, $R^{(22)} = \Lambda/L^{(1)}$.

For N>2 blocks with normal conditions at inner bounds ($\bar{u}^{(1)+}_n = \bar{u}^{(1)-}_{n+1}$, $\bar{\sigma}^{(1)+}_{11n} = \bar{\sigma}^{(1)-}_{11(n+1)}$), (2) results in

$$\sum\nolimits_{j=-1}^{1}(K^{(1)}_{nj}\bar{u}^{(1)}_{n+j} + K^{(2)}_{nj}\bar{u}^{(2)}_{n+j}) = 0,\ n = 1,..,N-1, \qquad (3)$$

where $K^{(1)}_{n,\pm1} = -\varphi^{\pm}_n\mu_{n\pm1}$, $K^{(1)}_{n,0} = {}^{(1)}r^2_n + (\varphi^+_n + \varphi^-_n)\mu_n$, $K^{(2)}_{n,\pm1} = -\varphi^{\pm}_n$, $K^{(2)}_{n,0} = \varphi^+_n + \varphi^-_n$, $\varphi^{\pm}_n = \dfrac{l_n}{l_n\eta_n + l_{n\pm1}\eta_{n\pm1}}$, $\eta_n = \eta({}^{(1)}r_n)$, $\mu_n = \mu({}^{(1)}r_n)$, $\eta(r) = \dfrac{N(r)}{M(r)}$, $\mu(r) = \dfrac{1}{M(r)}$, $l_n = 1/R^{(1)}_n$.

Chain of equations (3) (plus related equations resulting from external boundary conditions) is accurate, if one knows infinite number of initial values. If one knows only N_0 initial values for each TB, then operators $K^{(k)}_{nj}$ must be presented as series to order $O({}^{(k)}r^{N_0}_n)$ in vicinities of $N*N_0$ first exact eigenvalues, depending on external boundary conditions, most important of which are those at AF-DB boundaries, «switching» slip and stick modes and connecting DBs, the stresses at AF-DB, and other state parameters inside AF-DB. The resulting chain of shortened equations (3) also leads to accurate solutions for N^*N_0 values considered. Mass-averaged displacements only are considered in EPM models, so their solutions cannot be accurate for concrete boundary value problems. For instance, EPM models' solutions with N_0=2 have relative errors >=20% (!) for inner blocks.

3. Consider one TB with $V^{(1)\pm}=0$, then $u^{(1)}\equiv 0$ (=> $u\equiv u^{(2)}(y,t)$). Let $u^{-}\equiv 0$ and consider slip mode with $V^{+}=V_f(W)$, where $W\equiv\dot{a}=V_p-\dot{u}^{+}$, V_p ~0.1-15 cm/year is given mean velocity of another side of AF. Then (2) leads to equations: $\mathrm{ch}r\cdot u^{+}=M\cdot V_f(W)$, $M\overline{u}=Nu^{+}$ ($r={}^{(2)}r$), which solutions for linearized $V_f=V_{f0}-\gamma_f\dot{u}^{+}$ are $u^{+}=V_{f0}+\mathrm{Re}\sum A_k e^{r_k t}$, $\overline{u}=\dfrac{V_{f0}}{2}+\mathrm{Re}\sum A_k\dfrac{N(r_k)}{M(r_k)}e^{r_k t}$, where $r_k=-\lambda_f(\gamma_f)+i\omega_k$, $\lambda_f(\gamma_f)=\dfrac{1}{2}\ln\left|\dfrac{1+\gamma_f}{1-\gamma_f}\right|$, $\omega_k=\{\pi(1/2+\mathrm{k}),\ |\gamma_f|<1;\ \pi k,\ |\gamma_f|>1\}$. If $|\gamma_f|<1$ and only $u^{+}(0)$, $\dot{u}^{+}(0)$ are known, then shortened equation: $\dfrac{1}{\Omega^2}\ddot{u}^{+}+\dfrac{2\lambda_f}{\Omega^2}\dot{u}^{+}+u^{+}=V_{f0}$, $\Omega=|r_1|=\sqrt{\dfrac{\pi^2}{4}+\lambda_f^2(\gamma_f)}$ gives accurate solution $u^{+}(t)$. But solutions of connecting EPMs (for $u^{+}=2\overline{u}$): $\dfrac{1}{\tilde{\Omega}^2}\ddot{u}^{+}+\dfrac{2\gamma_f}{\tilde{\Omega}^2}\dot{u}^{+}+u^{+}=V_{f0}$, where $\tilde{\Omega}=\sqrt{k/m}=\mathrm{const}$, ($k$ and m – elasticity coefficient and mass), have infinitely large errors (!!) when $|\gamma_f|\to 1$.

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